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On the development of a general theory of the mechanics of tensile fracture of fibre reinforced materials

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A general analysis of the tensile fracture of fibre reinforced composites is proposed. It is applied here to the failure of laminates containing a notch or cut out of arbitrary size and also to the failure of a nominally undamaged unidirectional composite. Comparisons are made with published experimental data. In the case of laminates the model predicts the "hole size effect", the development of damage zones and the effect of various factors on notch sensitivity. The tensile strengths of unnotched unidirectional composites are also correctly predicted. The computation is based on the general form of the strain field surrounding a crack bridged by reinforcing members and has previously been applied in a modified form to transverse ply cracking in laminates. Since the analysis is based on a simplified physical description of the mechanics of fracture some insight is provided into the ways in which the mechanical characteristics of a composite system influence the failure processes and the stress levels at which they occur.

1. Introduction

Various types of failure processes are observed to occur in fibre reinforced materials subjected to a tensile load. A unidirectionally reinforced sheet loaded in tension perpendicular to the fibre direction fails by the growth of a crack propagating parallel to the fibres, i.e. at 90° to the loading direction. When the sheet is incorporated in a multiply laminate this failure process is inhibited by adjacent plies aligned at 0° and $\pm\theta°$ to the direction of loading.

An unnotched composite, in which all the fibres are aligned in the direction of loading, eventually fails by sequential fibre fracture and the separation of the fracture faces. This type of simple composite is not usually notch sensitive because, under load, the material splits parallel to the fibres at the ends of the notch. On the other hand laminates containing an appreciable proportion of 0° plies, together with plies in which the fibres are orientated at other angles to the direction of loading, are observed to be notch sensitive. In contrast with other materials the reduction in strength is not particularly sensitive to the shape of the notch or cut out but, within limits, is dependent on its absolute size. Damage zones, consisting of areas within which multiple cracking occurs, develop at the tips of notches in laminates and increase in size as the applied stress is increased. In the published literature the failure processes outlined above have been treated as independent phenomena and various analytical models have been developed in order to predict the stress levels at which the different types of failure may occur.

It has been proposed previously that the first of the failure processes mentioned, i.e. transverse ply cracking in laminates, could be described in terms of the strain field developed around the crack [1, 2]. In this paper the same basic model is developed to deal with the other failure processes mentioned, i.e. the tensile failure of notched angle ply laminates and unnotched unidirectional composites. Since the analysis is based on a simplified physical model of the mechanics of fracture some insight is provided into the ways in which the mechanical characteristics of a composite system influence the failure processes and the stress levels at which they occur.

When a simple unidirectional lamina, containing

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a transverse notch, is loaded in tension in the direction of the fibre alignment it is usual for shear cracks to develop at the ends of the notch and these propagate parallel to the direction of fibre alignment. The process is sometimes known as shear back. In this way a unidirectional 0° lamina becomes insensitive to the effect of the initial notch. If additional laminae oriented at other angles, e.g. ±45° and/or 90°, are added to the structure the laminate becomes notch sensitive, although the tensile strength of the laminate is still largely controlled by the strength of the 0° plies. The notch sensitivity is increased as the thicknesses of the individual laminae are decreased, and also depends on the proportion of fibres in the various plies in the laminate (see for example Taig [3]). It seems reasonable to suppose therefore that the notch sensitivity of a laminate, whose strength is largely set by the properties of the 0° plies, is controlled by the inhibition of shear back in the 0° plies due to the constraints exerted by the other plies. The presence of a notch reduces the stress supported by the 0° plies at laminate failure so that, as pointed out by Potter [4] it may be concluded that failure takes place by the sequential failure of the 0° fibres due to local stress concentrations generated by the notch. Various experimental studies have shown the strength reducing effect of a notch to be dependent on its size, when its overall dimensions are less than about 3 cm, but to be relatively insensitive to its shape.

A number of analytical approaches have been developed to predict the stress reducing effects of notches of various sizes. Waddoups et al. [5] assumed the existence of regions of intense energy, having a characteristic linear dimension, which extended symmetrically from each side of a hole perpendicular to the direction of the applied load. Local strains in excess of the general failure strain of the material were observed in the intense energy regions which were assumed to behave as cracks. A broadly similar approach was followed by Cruse [6] who considered that the size of a hole would be augmented by the presence of inherent flaws in the material. Whitney and Nuismer [7] proposed two criteria. In the first it was assumed that the strength of the unnotched material must be reached at a distance d_0 from the edge of the hole. In computing the stress values it was assumed that the plate containing the hole was isotropic and elastic. The value of d_0 was assumed to be a characteristic of the material and independent of laminate geometry or stress distribution. In the second criterion suggested by Whitney and Nuismer it was proposed that the average stress over a distance a_0 ahead of the discontinuity should equal the unnotched laminate strength. The value of a_0 was again assumed to be a material property independent of laminate construction and stress distribution. Thus, given the unnotched laminate strength and appropriate values for the critical distances a_0 or d_0 the strength of a laminate containing a hole or crack of arbitrary size could be predicted. Pipes et al. [8] developed an alternative analysis of the effects of a circular hole taking into account the orthotropic elastic properties of a laminate and used this to predict the notched strength of unidirectional boron–aluminium and various carbon fibre–epoxy resin laminates.

Potter [4] suggested that the hole size effect could be accounted for by the magnitude of the stress gradient in the laminate. He argued that the load carried by a fibre before fracture can be assumed to be transferred to the adjacent intact fibre. If the adjacent intact fibre is already carrying a high stress the additional load transferred to it by the broken fibre can cause it to fail. Thus the possibility of sequential fibre failure depends on the initial stress gradient in the laminate. It was possible to account for the observed decrease in strength with increasing hole size for 0° ± 45° carbon fibre laminates using a theoretical analysis based on this argument. Further experimental studies of the notched strength of carbon fibre laminates has been carried out by Peters [9] and by Ochiai and Peters [10]. Caprino [11] has applied recently a two-parameter model, based on an intrinsic flaw concept, to various laminates.

The various approaches which have been developed to predict the strength of notched laminates are all concerned with the sequential fracture of the 0° load bearing fibres. However, quite different analyses have been developed to deal with the failure of a unidirectionally aligned unnotched fibre composite loaded in tension in a direction of the fibre alignment although the failure process is again considered to be due to the sequential failure of 0° fibres. Since the fibres contain flaws of varying severity distributed randomly along their lengths, fibre failure takes place initially at the positions of the most severe flaws. Load is transferred back to the broken fibres from the matrix so that the load bearing ability of the fibre in the composite is maintained at some little distance from the point of failure. This stress transfer distance can be estimated. A considerable amount of work has been carried out on this topic (see Harlow and Phoenix [12] for a review). Calculations have been made of the additional stresses transferred to the fibres adjacent to a broken fibre or fibres [13]. The local stress enhancement increases as the number of broken fibres in the group increases so that the likelihood of catastrophic failure by sequential fibre fracture increases as the size of the cluster of broken fibres increases. According to this analysis the number of broken fibres in a cluster of critical size will be a function of the detailed characteristics of the composite structure but will generally be less than ten [14]. A broadly similar analysis, which also encompasses dynamic stress concentration effects following the failure of an individual fibre, has been developed by Barry [15]. Barry also carried out a systematic investigation of the tensile strengths of various types of carbon fibres and their composites [16], [17].

An analysis of the mechanics of the growth of a crack in a matrix bridged orthogonally by reinforcing fibres has been proposed previously [18]. The analysis is based on the form of the strain field surrounding the crack. This model enables the critical stresses for the growth of cracks of arbitrary length to be calculated from a computation of the rates of release and absorption of energy with increasing crack length and/or increasing applied stress. Recently the model has been applied to $\pm 0^\circ/90^\circ$ laminates [1]. A further modification of this model has been developed to deal with the mechanics of growth of a matrix crack when some of the crack bridging fibres have fractured [2] and the same basic argument can also be applied to hybrid systems containing more than one type of fibre [19]. The validity of the strain field model has been supported directly by measurements of the strain field surrounding a crack bridged by reinforcing members [20] and indirectly through the agreement obtained between experimental and calculated stresses for the growth of a matrix crack bridged by reinforcing members [1, 2, 18].

In this paper a further modification of the strain field model has been developed to deal with the failure of $0^\circ/\pm\theta^\circ$ and $0^\circ/90^\circ$ laminates containing notches and cut-outs. A transverse crack is assumed to grow from the edge of a cut-out in the 0° plies thus increasing the length of the initial notch. In the interests of simplicity, failure by forward shear (shear back) in the 0° ply is assumed to be suppressed completely. The analysis therefore deals with the lower bound of the strengths of damaged laminates. The form of the strain field surrounding the total crack is based on the models previously developed and numerical values for the increase in the length of a transverse stable crack in the 0° ply with increasing stress are obtained from computations of the rates of release and absorption of strain energy with increasing crack length. The model thus conforms with the observed development of damage zones at the tips of cracks in laminates. The extending crack is bridged by reinforcing fibres which eventually fail due to the increasing stress which they carry and this initiates unstable crack growth and failure of the laminate. The stress at which this occurs decreases as the size of initial notch increases and corresponds with the experimentally observed "hole size effect".

By use of the same model the strength of nominally unnotched unidirectional specimens is obtained by computing the stress at fracture when the size of the initial notch has been reduced to dimensions corresponding to expected adventitious damage in a practical composite. The model successfully predicts the observed tensile strengths of unidirectional composites for these conditions.

It is therefore suggested that the strain field model can be used to calculate the tensile strengths of unidirectional composites containing either a single type of fibre or a mixture of different fibres; the minimum strengths of laminates constructed with a significant fraction of 0° fibres and containing a cut-out of arbitrary size; and the transverse ply cracking strain for $0^\circ/90^\circ$ laminates.

2. The analytical model

2.1. General outline of the strain field model

The analysis developed in this paper is based on a model previously proposed [1, 2, 18], which describes the strain field around a matrix crack bridged by reinforcing fibres.

The general form of the strain field is shown in Fig. 1. A unidirectionally reinforced composite is assumed to be subjected to a uniform tensile strain ϵ_β generated by a load applied in the direction of fibre alignment. This is perturbed within an elliptical zone around the matrix crack. Elastic strains

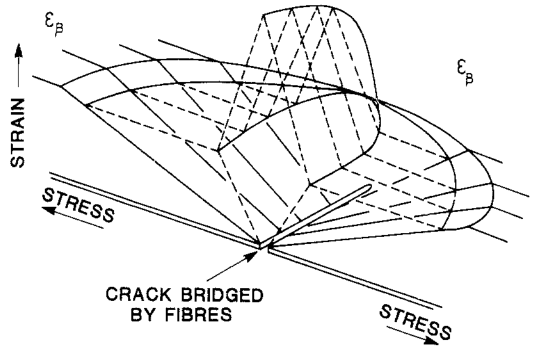

Figure 1 Schematic illustration of one-half of the assumed strain field developed around a matrix crack bridged by reinforcing fibres. Strain values plotted on the vertical axis. Full lines indicate the strain field which would be developed in the absence of the reinforcing fibres.

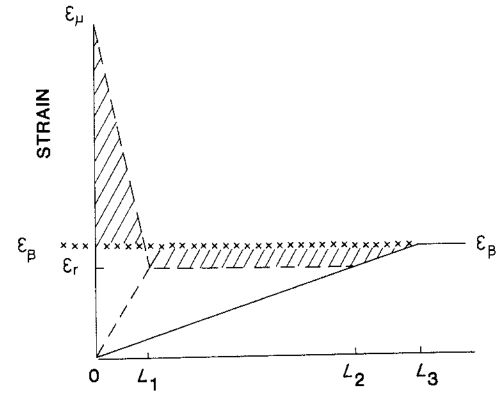

Figure 2 Section through a quadrant of the strain field. Uniform strain in composite with uncracked matrix indicated thus X X X X. Shaded areas indicate the amount of fibre extension and contraction generated by the matrix crack.

developed in the fibres and the matrix are plotted on the vertical axis in Fig. 1. In the absence of any reinforcing fibres the matrix strain is assumed to increase linearly in the direction of the applied load from zero at the crack faces to ϵ_β at the edge of the elliptical zone. The zone is considered to be divided into a number of independent parallel segments. Strains are assumed to be developed only in the direction of loading and on this basis the strain energy released by each segment and hence by the relaxation of all of the material within the elliptical zone is readily calculated. When the major axis of the zone is three times the crack length the strain energy released (calculated on the basis of the assumptions made above) is numerically equal to the value computed by Griffith [21] by integrating the strain field around a crack in an isotropic elastic material computed from elasticity theory. When the fibre volume fraction is zero the model developed here therefore predicts the same numerical value for the critical crack length as that obtained from the Griffith analysis.

The effect on the strain field of the presence of crack bridging reinforcing fibres aligned perpendicularly to the crack faces is computed assuming a constant shear stress transfer value, τ, at the fibre matrix interface. The fibres carry their maximum load where they bridge the crack and this additional load is transferred from the fibres to the matrix progressively with increasing distance from the crack face. The strain carried by the fibres therefore falls with increasing distance from the crack face and that carried by the matrix increases over its unreinforced condition. The strain distribution between fibres and matrix within a segment of the composite parallel to the fibre alignment and loading direction is shown in Fig. 2. At some distance from the crack face, defined as L_1, the strains carried by fibres and matrix are equal and it is assumed that the strain distribution at greater distances from the crack face takes the form indicated in Fig. 2. The general validity of this assumption has been confirmed from experimental observations of the strain field developed round a crack bridged by reinforcing fibres [20]. The relationships between the various quantities indicated in Fig. 2 are as follows:

$$\begin{aligned} \epsilon_r &= \{\epsilon_\beta L_3/[Q(P+\epsilon_\beta/L_3)^{-2} + L_3/\epsilon_\beta]\}^{1/2} \\ \epsilon_\mu &= L_1(P+Q+\epsilon_\beta/L_3) \qquad (1) \\ L_1 &= \epsilon_r(P+\epsilon_\beta/L_3)^{-1} \\ L_2 &= L_3\epsilon_r/\epsilon_\beta \qquad L_3 = 3(a^2-z^2)^{1/2} \end{aligned}$$

where $P = 2V_f\tau/E_mV_mr$, $Q = 2\tau/E_fr$, and z defines the distance of the section considered from the centre of the crack. V_f and V_m are the fibre and matrix volume fractions respectively and $2r$ is the fibre diameter. The strain energy released by a parallel sided strip of width δz and of unit thickness positioned at a distance z from the centre of the crack is given by δW_{Rz} where

$$\begin{aligned} \delta W_{Rz} = \{&E_c\epsilon_\beta^2L_3/2 - E_c\epsilon_\beta^2(L_3^3-L_2^3)/6L_3^2 \\ &- E_c\epsilon_r^2(L_2-L_1)/2 - E_m\epsilon_r^2L_1/6 \qquad (2) \\ &- V_fE_f(\epsilon_\mu^2+\epsilon_\mu\epsilon_r+\epsilon_r^2)L_1/6\}\delta z \end{aligned}$$

E_f and E_m are Young's modulus values of fibres and matrix respectively and the other symbols have their usual meanings. The strain energy released by the whole of the zone can be obtained by numerical integration. The rate of release of energy with increasing crack length can then be obtained by numerical differentiation [18].

In the form described above the analytical model considers only the mechanics of the growth of a matrix crack bridged by fibres which remain intact having a higher failing strain than the matrix. The model has been modified to deal with fibres having non-uniform strengths so that some of the crack bridging fibres fracture at stresses below that at which the matrix crack becomes unstable [2]. The proportion of fibres which fracture depends upon the distribution of fibre failing strains and the strains computed for the crack bridging fibres. The fibres which have fractured, whilst the matrix crack is still stable, are assumed to become part of the matrix and consequently increase its elastic modulus. Unstable crack growth occurs at a critical stress level by the sequential failure of the remaining intact crack bridging fibres.

Strain energy is absorbed in rupturing the matrix and for the purposes of calculation, a transverse planar crack is assumed to be developed at failure. Energy is also absorbed in rupturing the chemical bond between the fibres and the matrix and by frictional losses developed by differential movement between the crack bridging fibres and the matrix [2].

The energy absorbed by frictional losses within width δz and of unit thickness, within the elliptical zone around the crack is given by [18]

$$\delta W_{Az} = V_f \tau \epsilon_\mu L_1^2 \delta z / 3\tau \quad (3)$$

If the work done in debonding a unit area of fibre/matrix interface is G_d the work done on each side of the crack per unit increase in crack area will be given by [2]

$$4 V_f L_1 G_d / r \quad (4)$$

The presence of the crack bridging fibres serves to reduce the rate of release of strain energy with increasing crack length compared with the unreinforced material. Also the rate of absorption of energy increases with increasing crack length because of the additional crack length dependent energy absorbing processes discussed above. Hence the stability of a matrix crack is enhanced considerably by the presence of the fibres. As the applied load is increased the model predicts that the crack will initially increase in length but remain stable even though some of the weaker crack bridging fibres may fracture. Eventually at a sufficiently high strain level the crack becomes unstable because of the sequential failure of the remaining crack bridging fibres. The model assumes that the fibres all fracture in the plane of the crack. In many cases this is not so and short lengths of fibres are extracted from the matrix from each side of the primary zone of fracture. In these cases energy is absorbed by fibre pull-out but, according to the model used here, this occurs at a diminishing stress level after the unstable extension of the primary crack. Hence energy absorbed by fibre pull-out is not taken into account in the crack stability calculations. It will be noted that the primary mechanism governing crack growth is the catastrophic failure of the crack bridging fibres.

2.2. Analysis of notch induced transverse crack growth

In the case of a unidirectional lamina containing a notch or cut out of appreciable size and loaded in tension in the direction of fibre alignment, cracks usually propagate from the ends of the notch parallel to the fibres (Fig. 3). This process prevents the development of a transverse crack from the notch tip so that the lamina is notch insensitive from the point of view of transverse crack growth. The Griffith analysis shows that, for an isotropic elastic sheet, the stress required for transverse crack growth is given by $(EG_1/\pi a)^{1/2}$ where $2a$ is the initial crack length and G_1 the work of fracture associated with the crack opening mode of failure. The stress required for fracture by forward shear from the ends of notch in the direction of the applied load can be calculated from elementary considerations as $(2EG_{11}/a)^{1/2}$ where G_{11} is the work of fracture associated with crack growth by forward shear. In the case of a homogeneous elastic solid transverse crack growth is energetically more favourable. This is not generally so in the case of a unidirectionally reinforced lamina because of the anisotropic characteristics of the material. Transverse crack growth can however take place in the 0° plies of multiply laminates if the forward shear (shear back) mode of failure is suppressed in the 0° plies by the presence of adjacent $\pm\theta°$ plies and 90° plies. The extent to which shear back is suppressed will depend on the

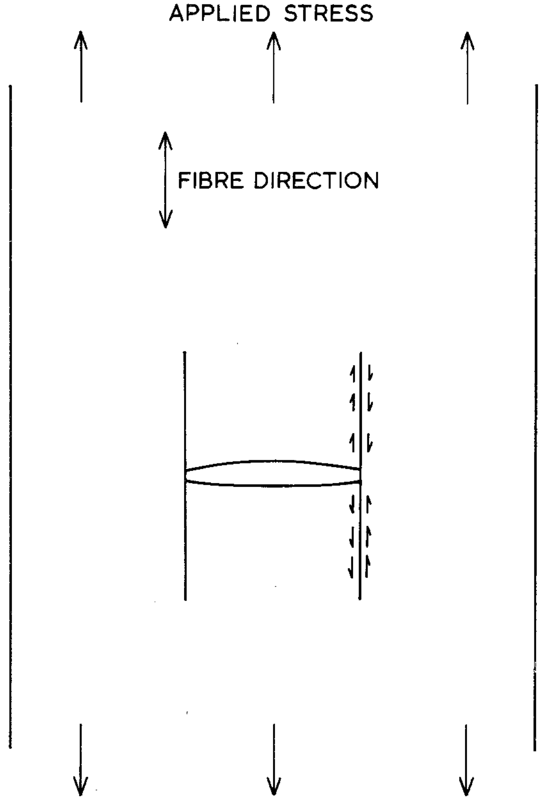

Figure 3 Illustrating transverse crack blunting by shear back.

details of the laminate construction. In the analysis below it is assumed that complete suppression of shear back occurs. This situation corresponds to the maximum notch sensitivity of the 0° plies and thus sets a lower limit to the strength of a damaged laminate.

In the case of nominally unnotched unidirectional fibrous composites any existing flaw can be assumed to be due to adventitious damage and can therefore be assumed to have dimensions not greater than ~ 1 mm. In most types of carbon fibre polymer matrix composites the tensile fracture faces are more or less planar with a limited amount of fibre pull-out and it seems reasonable to suppose that the failure process is dominated by transverse crack growth. Thus it would seem that the analysis developed below should be applicable to the tensile failure of nominally unnotched unidirectional carbon fibre composites providing the size of the initial notch is set at a value corresponding to that which would be expected as a consequence of adventitious damage.

The general features of the model used are illustrated in Fig. 4. A transverse crack of total length $2C$ is assumed to have propagated from the ends of a cut-out of length $2A$. The elliptical region around the crack, within which the general strain ϵ_β carried by the laminate as a consequence of the applied tensile load is perturbed, is indicated. The length of the semi major axis of this zone is $3C$. Since no fibres traverse the cut-out the elastic relaxation of the matrix and the fibres within the zone of width $2A$ on each side of the crack (shown shaded) is resisted by the intact fibres bridging the distance $(C-A)$ on each side of the initial cut-out.

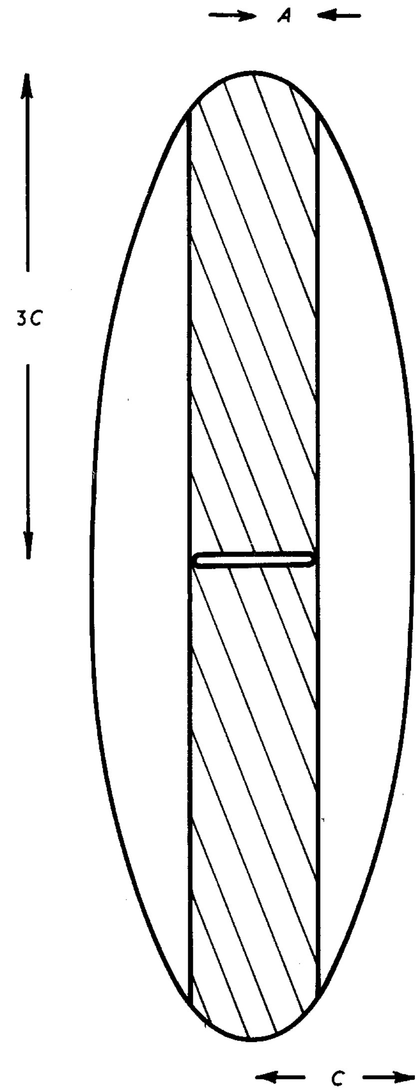

Figure 4 Illustrating development of a transverse crack from an initial notch showing the region within which the strain field is modified.

No attempt has been made in the analysis to describe in detail the strain distribution within the

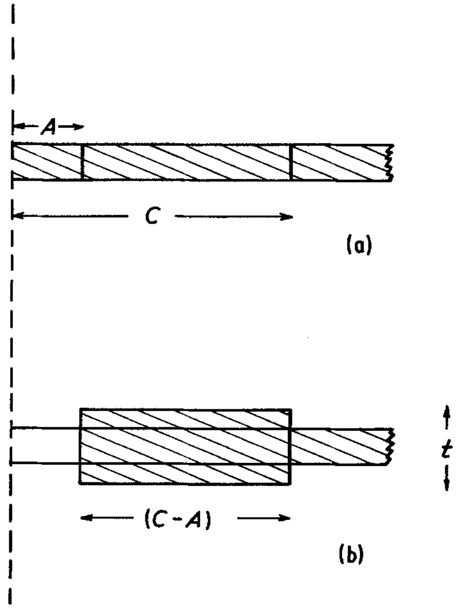

Figure 5 Section through one-half of the crack illustrating the equivalent physical model used in the analysis.

shaded zone shown in Fig. 4. When $(C-A)$ is small the strain field round the notch must approximate to that derived from classical elasticity theory. When $C \gg A$ the strain field approaches that of a crack bridged by a uniform distribution of fibres [2]. The conditions pertaining to the development of stable cracks concern intermediate situations and the model represents these approximately. The crack length increases very rapidly with increasing strain at the onset of instability so that for this condition $C \gg A$. In the analysis used here an attempt has been made to produce a simple analytical model which gives an approximate representation of the physical situation. The general validity of the model is discussed in subsect. 2.3.

In the analysis it is assumed that the strain carried by the material in the central zone, width $2A$ (in which all the crack bridging fibres have fractured) corresponds to that of the matrix within the zones extending from A to C. The equivalent physical model envisaged is illustrated in Fig. 5 in which a cross section of one-half of the crack in the lamina is shown. Here the material contained within the central sector (Fig. 5a) is assumed to be redistributed on each side of the lamina over the distance $C-A$ (Fig. 5b). The lamina is assumed to have unit thickness so that the thickness of the material, t, over the distance $C-A$ is $C/(C-A)$. The total thickness of the two layers of material redistributed over the distance $C-A$ is given by $A/(C-A)$. Thus the relative thickness of the outer layer falls as C increases.

The central layer of the sandwich (Fig. 5b) contains reinforcing fibres which bridge the crack extending over the distance $(C-A)$. A proportion of these crack bridging fibres will fracture depending on the stresses applied and the distribution of fibre strengths. We denote the total volume fraction of fibres in the central layer by V_{ft} and the proportion which have fractured as NV_{ft}. Following a previous analysis [2] the fibres which have fractured are regarded as part of the matrix and enhance its elastic modulus. Within the central zone, defined by the distance A in Fig. 5a, all the fibres have fractured so that the elastic modulus of the material in this region is given by

$$E_p V_p + V_{ft} E_f \qquad (5)$$

and this then corresponds to the material in the outer layers of the sandwich shown in Fig. 5b. E_p is the elastic modulus of the polymeric binder and V_p its volume fraction. The elastic modulus of the fibres is given by E_f. Within the central zone of the model (Fig. 5b) within which crack bridging fibres are present, the elastic modulus of the matrix will be given by

$$E_p V_p + NV_{ft} E_f \qquad (6)$$

In calculating the average elastic modulus of the matrix in the inner and outer layers (Fig. 5b) these expressions have to be factored for the relative thicknesses of the layers. Hence over the distance $C-A$ the matrix elastic modulus is given by E_m where

$$E_m = (E_p V_p + NV_{ft} E_f)(C-A)/C + E_p V_p + V_{ft} E_f (A/C) \qquad (7)$$

Crack bridging fibres are present only in the central layer and their volume fraction within that layer is given by

$$(1-N)V_{ft} \qquad (8)$$

Hence the fibre volume fraction within the total sandwich of thickness t (Fig. 5b) is given by V_f where

$$V_f = (1-N)V_{ft}(C-A)/C \qquad (9)$$

The volume fraction of the matrix V_m is of course given by $V_m = (1 - V_f)$.

In reality the matrix crack is propagating only in the central layer of the sandwich so that the

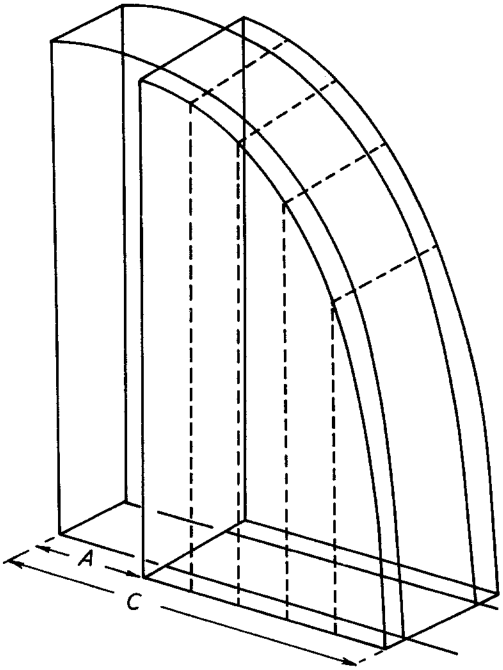

Figure 6 Illustrating the physical model used within one quadrant of the zone.

work of fracture G is given by

$$G = (V_p W_{pf} + NV_{ft} W_{ff}) \quad (10)$$

where W_{ff} is the work of fracture of the fibres and W_{pf} is the work of fracture of the polymeric binder.

In order to compute the rate of release and absorption of strain energy with increasing crack length the elliptical zone around the crack has been divided into five segments. One quadrant of the zone is shown in Fig. 6. Each of the segments has a width equal to KA where K is an arbitrary constant so that the total matrix half crack length C is given by

$$C = A + 5KA \quad (11)$$

Thus the crack length C is increased by increasing the value of the arbitrary constant K.

The distance from the centre of the crack to the centre line of each segment is given by

$$A + FKA \quad (12)$$

where $F = 0.5, 1.5, 2.5, 3.5$ and 4.5. The length of a segment in one quadrant of the elliptical zone is given by L_3 (see Fig. 2) where from the properties of an ellipse,

$$L_3 = \{C^2 - (A + FKA)^2\}^{1/2} \quad (13)$$

From Equation 2 the strain energy released per unit cross sectional area of composite can be obtained. If this is denoted by WS then the strain energy released by one segment of the zones described in Fig. 6 is given by $WSKAC/(C-A)$, since the width of a segment is given by KA and its thickness by $C/(C-A)$. The strain energy released within two quadrants of the elliptical zone is thus computed by numerical integration and the incremental change in strain energy ΔU for an incremental increase in crack length, ΔC, computed by numerical differentiation. Thus the rate of release of strain energy with increasing crack length $\Delta U/\Delta C$ is derived and compared with the corresponding rate of absorption of strain energy, $\Delta G/\Delta C$. Note that the actual crack front in the lamina within which the rate of absorption of energy by matrix rupture is computed is defined by the width of the central element of the sandwich shown in Fig. 5b.

The computation of the length of a stable crack as a function of the applied strain ϵ_β is obtained in the following manner for an initial crack of arbitrary length $2A$. A low initial value is first taken for K so that C is slightly larger than A (Fig. 4). The values of $\Delta U/\Delta C$ and $\Delta G/\Delta C$ are then computed and if $\Delta U/\Delta C > \Delta G/\Delta C$ (as will be the case initially) the value of K and hence C is increased and the cycle of computation repeated. Eventually $\Delta U/\Delta C < \Delta G/\Delta C$ so that the matrix crack is stable. The value of ϵ_β is then increased and the cycle of computation repeated to obtain the corresponding stable matrix crack length at the enhanced value of ϵ_β.

During the computation of stable crack lengths the value of the peak tensile strain carried by the crack bridging fibres in the first segment immediately adjacent to the inital flaw of half-length A is recorded. When this value reaches the lower limit of the assumed distribution of fibre failing strains fracture of some of the crack bridging fibres is predicted. This modifies the characteristics of the system and causes the length of the stable matrix crack to increase more rapidly with increasing values of ϵ_β. The rate of increase of the crack length with increasing values of ϵ_β eventually becomes infinitely large indicating unstable crack extension.

2.3. Comments on the validity of the analytical model

One problem in the development of an analytical model capable of describing adequately the mechanics of the growth of a matrix crack from an existing notch is the need to consider the effect of the non-uniform elastic relaxation which must take place within the zone around the notch shown shaded in Fig. 4. In the formulation of the model it is assumed that the material (fibres plus matrix) within the shaded zone would relax elastically by the same amount as the matrix material in the immediate vicinity of the crack bridging fibres over the crack length $(C-A)$. Clearly this is a poor representation of the situation when $(C-A)$ is small and for these conditions the material near the centre of the initial notch must relax by an amount considerably greater than the assumed value. The model does not properly take account of this effect. The cross-sectional areas of the outer layers of the sandwich and that of the material within the plane of the initial notch are equal but the lengths of the of the strips forming the various segments of the elliptical zone are different (Fig. 6). The model thus underestimates the volume of material assumed to be transferred from the shaded central zone, Fig. 4, to form the outer layers of the sandwich (Fig. 6). Thus, when $(C-A)$ is small the model underestimates the amount of strain energy released. In this context it is interesting to note that the strain values at which transverse crack growth from the notch are calculated to occur from this analysis are in reasonable agreement with the critical strain values for unstable crack growth predicted by the Griffith equation taken, for the latter condition, as an isotropic homogeneous elastic material in which the resistance to crack growth is provided by the work of fracture of the polymeric matrix.

When $C \gg A$ the model still underestimates the volume of material transferred from the central zone shaded in Fig. 4, to form the outer layer of the sandwich (Fig. 6), but this is now small compared with the total volume of material within the elliptical zone. The transverse crack becomes unstable when $C \gg A$ and the primary factor causing instability is the commencement of fracture of the weaker crack bridging fibres. Within its overall limitations therefore it is felt that the model gives a reasonable approximation to the conditions pertaining to the onset of instability in the growth of a transverse crack.

Clearly it would be possible to develop a more comprehensive analysis of the mechanics of transverse crack growth than the simple model presented here. However, this would be at the expense of greater complexity and it would be difficult to confirm the validity of such a model in view of the lack of relevant experimental data.

3. Results of calculations

Transverse crack growth calculations have been performed on two types of unidirectional lamina all containing a fibre volume fraction of 60%. The fibres were representative of types II and III carbon fibres and the matrix characteristics corresponded to those of a typical epoxy resin. Details of the systems investigated are shown in Table I, the numerical values being based on data provided by Barry [16, 17]. The fibre failing strains are assumed to be distributed uniformly between upper and lower values, ϵ^*_{max} and ϵ^*_{min}. Potter [4] has provided data on the reduction in strength of $\theta° \pm 45°$ laminates containing holes of various sizes constructed from type II fibres and Sturgeon [22] has carried out tensile and fatigue tests on unidirectional and $0° \pm 45°$ laminates constructed from type III carbon fibres. Comparisons are made between these data and the results of the calculations carried out here.

3.1. Damage zone development

The analysis developed here predicts that transverse growth from a notch will be initiated at some critical strain value which increases as the notch

TABLE I Systems studied

Fibre volume fraction	0.6
Fibre diameter	10 μm
Fibre work of fracture	150 Jm^{-2}
Matrix elastic modulus	4 GPa
Matrix work of fracture	200 Jm^{-2}
Elastic modulus (type II fibres)	245 GPa
Maximum fibre failing strain (type II fibres) ϵ^*_{max}	0.0163
Minimum fibre failing strain (type II fibres) ϵ^*_{min}	0.01225
Elastic modulus (type III fibres)	200 GPa
Maximum fibre failing strain (type III fibres) ϵ^*_{max}	0.01875
Minimum fibre failing strain (type III fibres) ϵ^*_{min}	0.01375

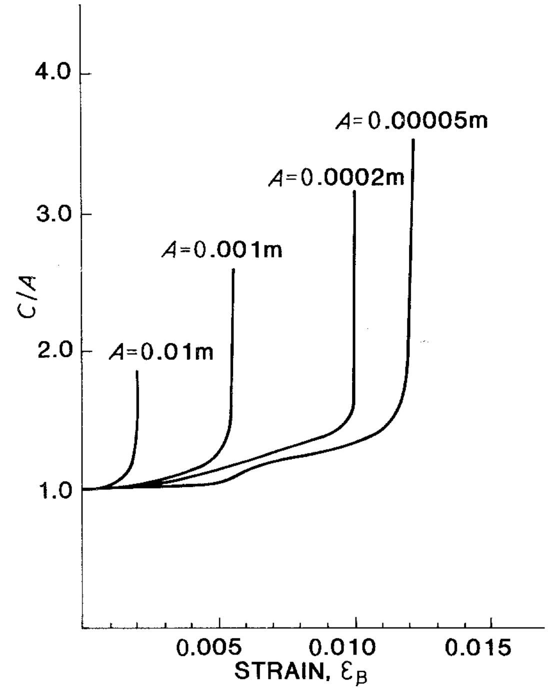

Figure 7 Illustrating the growth of a transverse crack with increasing composite strain from notches of various sizes. $E_f = 245$ GPa, $V_f = 0.6$, $\tau = 10$ MPa, $G_d = 50$ Jm^{-2}.

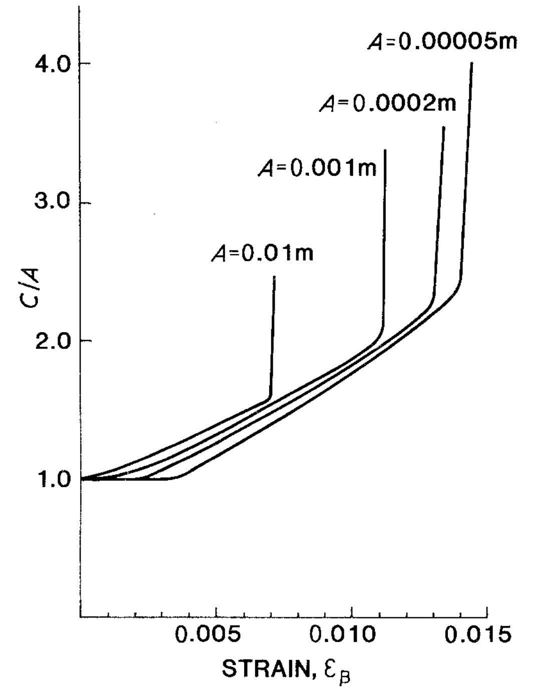

Figure 8 Illustrating the growth of a transverse crack with increasing composite strain from notches of various sizes. $E_f = 200$ GPa, $V_f = 0.6$, $\tau = 1$ MPa, $G_d = 15$ Jm^{-2}.

size decreases. The transverse crack is stable and increases in length as the composite strain increases. Data corresponding to notches of various sizes (0.01, 0.001, 0.0002 and 0.00005 m) for two types of carbon fibre composites are shown in Figs. 7 and 8, the ratio of transverse crack length/initial notch length being plotted against the composite tensile strain ϵ_β. The data indicate a gradual increase in the length of the stable transverse crack with increasing strain followed by a rapid increase in crack length indicating unstable crack extension. The change in slope of the crack length versus composite strain curve occurs when the strain carried by the crack bridging fibres is sufficient to cause those having low strengths to fail.

The analysis given here is confined to the growth of a single transverse crack in the 0° plies of a laminate. However it is apparent that the local deformations associated with this process would be expected to initiate cracking parallel with the fibres in the ±45° plies. Also the network of cracks formed in this way would be expected to increase in number and extent with the progressive increase in length with increasing strain of the primary transverse crack. The model is therefore consistent with the development of damage zones widely observed to be initiated at the tips of notches in laminates. The model is also consistent with the observation by Waddoups *et al.* [5] of strains in excess of the laminate failing strain at and near the tip of a notch. The model predicts that a transverse crack can remain stable even though the local strains in this region are sufficiently large to cause extensive fibre fracture.

3.2. Laminate strength against notch size

It is apparent from Figs. 7 and 8 that the laminate strain at which unstable crack growth occurs falls as the size of the initial notch increases. Curves similar to those shown in Figs. 7 and 8 were constructed for laminates reinforced with various types of carbon fibres and the calculated strains for unstable crack growth are presented in Figs. 9 and 10. These figures show that the strength of a lamina falls progressively with increasing notch size and that the notch sensitivity increases as the coupling between the fibres and the matrix increases.

In Fig. 10 the experimental data obtained by Potter [4] for 0° ± 45° laminates constructed from type II carbon fibres are shown. These data

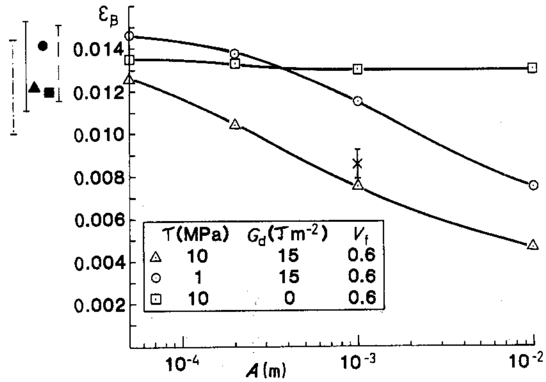

Figure 9 Illustrating the effect of notch size on the failing strain of various laminates. $E_f = 200$ GPa. Corresponding values for $V_f = 0.2$ and $A = 0.00005$ m shown ● ▲ ■. Data from Barry [17] shown . (Average values ± two standard deviations.) Calculated from data given by Sturgeon [22] shown (unnotched) and (2 mm diameter hole) — average values ± two standard deviations.

lie near the curve corresponding to interfacial decoupling energies of 50 $\mathrm{J\,m^{-2}}$ and values of τ of 10 MPa. Potter [4] and Sturgeon [22] noted that the notch sensitivity of the $0^0_2 \pm 45^\circ$ laminates depended upon the surface treatment which had been given to the fibres in order to improve their adhesion to the matrix and this effect is predicted by the model. It should be noted that the model, in the form developed here, deals only with the growth of a transverse crack in the 0° ply and assumes that shear back has been suppressed completely. It might be expected that strongly bonded fibres in the $\pm\theta^\circ$ plies would suppress shear back in the 0° plies more effectively so that

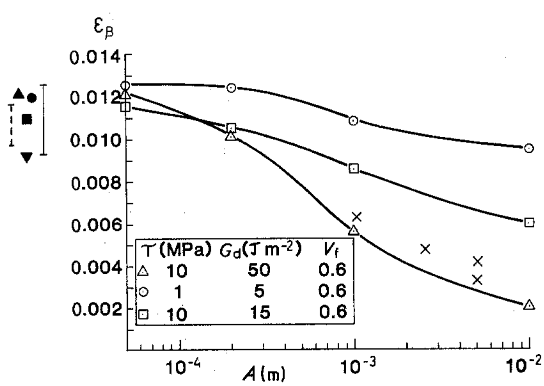

Figure 10 Illustrating the effect of notch size on the failing strain of various laminates. $E_f = 245$ GPa. Corresponding values for $V_f = 0.2$ and $A = 0.00005$ shown ● ▲ ■. Data from Barry [17] shown thus . (Average values ± two standard deviations.) Data from Potter [4] shown X for laminates with holes and ▼ for unnotched composite.

this mechanism could also contribute to the notch sensitivity of the 0° plies.

One further point of significance emerges from the observations of the effect of carbon fibre surface treatment on notch sensitivity. When fibres are extracted from the matrix by pull out it is clear that interfacial debonding must have occurred at some stage of the fracture process. In the case of the analytical model used here interfacial debonding is associated with the development of a matrix crack and must occur before the matrix crack faces can separate. The calculations indicate increasing notch sensitivity with increasing levels of chemical bonding so that the model is consistent with the experimental data. However, in order that the model should describe accurately the physical process of failure the matrix failing strain in the composite must be less than the failing strain of the fibres. This is generally the case for glass fibre reinforced polymer. In the case of many carbon fibre polymer matrix systems the fibre failing strains are less than those of the polymeric matrix when the components are tested independently. However, when in the form of a composite, mechanical constraints exerted by the fibres will tend to reduce the failing strain of the matrix.

3.3. Strength of unnotched specimens

In Figs. 9 and 10, the strength of the unidirectional composite is plotted for notches of dimensions decreasing to sizes comparable with individual fibres. It is to be expected that nominally unnotched specimens will contain some degree of surface and perhaps internal damage. It is suggested that damage of this type could be modelled as notches of various dimensions from local clusters of a few fractured fibres up to surface notches of the order of a few tenths of a millimetre in depth.

It is interesting to note that the range of composite strengths predicted by the model for notches of these dimensions correspond closely with the observed failing strains of nominally unnotched unidirectional carbon fibre composites. The data of Potter [4] and Sturgeon [22] refer to composites having a nominal fibre volume fraction of 0.6. Computed failing strain values for a fibre volume fraction of 0.2 and a notch length, A, of 0.00005 are also shown together with data from Barry [17] obtained on composites having fibre volume fractions ranging from 0.2 to 0.25. It is apparent that, on the basis of the model used here the composite failing strains are relatively insensitive to fibre volume fraction. The model is not applicable to notches having dimensions less than several fibre diameters but shows strength levels diminishing for notch sizes greater than these values. The results of the analysis are therefore generally consistent with the findings of Phoenix and co-workers [12, 14], who computed from the chain of bundles theory that the onset of instability in unnotched composites would be reached when the number of broken fibres in a cluster was of the order of ten.

3.4. Discussion

It is suggested that the strain field theory of fracture can be used to predict the tensile strengths of fibrous composites and laminates under the following conditions. These are (a) nominally unnotched unidirectionally reinforced composites, (b) the lower limit for the strength of $0^\circ \pm \theta^\circ$ laminates containing holes and cut-outs of arbitrary size, and (c) the cracking strains of the transverse plies of $0^\circ/90^\circ$ laminates. Experimental support for the assumed form of the strain field used in this analysis has recently been obtained [20]. The former two failure processes have been discussed in this paper, the latter elsewhere. See [1, 2].

The analysis of the effect of notches and cut-outs in $0^\circ \pm \theta^\circ$ laminates developed here correctly predicts the strength of the laminate to be dependent on the absolute size of the notch. It is assumed that transverse matrix cracks will propagate readily from the tips of a pre-existing notch. For these conditions the laminate strength would be expected to be relatively independent of the shape of the notch since the notch shape only modifies slightly the volume of material around the crack which has relaxed elastically to provide the energy for crack propagation.

In the analysis given here forward shear (shear back) is assumed to be completely suppressed so that the 0° ply is constrained to fail by the propagation of a transverse crack. For these conditions the notch sensitivity of the laminate is calculated to increase as the coupling between the fibres and the matrix is increased as is observed experimentally. Good agreement between theory and experiment is obtained when realistic values for the chemical bonding of the fibre matrix interface and the residual frictional shear strength of the interface after debonding are inserted into the analysis. The analysis thus enables the effects of changes in the coupling between fibres and matrix to be predicted.

The model is also consistent with the observed decrease in the notch sensitivity of a laminate after cyclic loading. It may be assumed that shear back occurs progressively in the 0° plies during cyclic loading as part of the development of damage zones.

The notch sensitivity of $0^\circ/\pm 45^\circ$ laminates is observed to fall as the thicknesses of the individual plies is increased. The effect is caused by enhanced interply delamination which reduces the interaction between plies [23]. The model is consistent with this observation since delamination allows shear back to take place in the 0° plies thus reducing their notch sensitivity.

When the notch size is reduced to very small dimensions the model predicts composite tensile strength values corresponding to those observed for unidirectional composites. Also, the variability in the observed strength measurements can be ascribed, according to the model, to various degrees of relatively minor surface or internal damage to the composite. According to the analysis the notch sensitivity of the nominally unnotched unidirectional composites is not influenced significantly by the interaction between the fibres and the matrix. This is not the case when the notch is of appreciable dimensions. The approximate form of the strain field surrounding a crack bridged by reinforcing elements, which forms the basis of the analysis developed here, seems to provide an adequate description of the true strain field. The analysis is capable of improvement from further direct observations of the strain fields developed in various physical models. A primary advantage of the model is the direct link it provides between the physical parameters of a composite system and the energetics of crack growth.

The analysis is open to further modification in order to deal with composite systems in which the matrix has a higher failing strain than the fibres. The crack can be assumed to be formed by fractured fibres and to be bridged by intact fibres plus the matrix. It may also prove possible to apply the strain field analysis to the development of damage zones in laminates under zero tension cyclic loading. However, very different analytical models would be required to deal with the micro-mechanics of failure under compressive or shear loading.

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